# The complexity of the stock market

"... a web of interrelated return effects."

Bruce I. Jacobs and Kenneth N. Levy

nvestment theory and practice have evolved rapidly and tumultuously in recent years. Many placed the Efficient Market Hypothesis (EMH) and the Capital Asset Pricing Model (CAPM) on pedestals in the 1970s, only to see them come crashing down in the 1980s. In explaining why such theories cannot represent the true complexity of security pricing, we suggest new approaches to coping with the market's complexity. To do so, we follow a taxonomy from the sciences.

Scientists classify systems into three types ordered, complex, and random.<sup>1</sup> Ordered systems are simple and predictable, such as the neatly arranged lattice of carbon atoms in a diamond crystal. Similarly, Newton's Laws of Motion are a simple set of rules that accurately describe the movement of physical objects. At the other extreme, random systems are inherently unpredictable; an example is the random behavior, or Brownian Motion, of gas molecules.

Complex systems fall somewhere between the domains of order and randomness.<sup>2</sup> The field of molecular biology exemplifies complexity. The mysteries of DNA can be unraveled only with the aid of computational science. The human mind alone cannot cope with DNA's complexity, nor do simple theories suffice.

The stock market, too, is a complex system.<sup>3</sup> Security pricing is not merely random, nor are simple theories adequate to explain market operation. Rather, the market is permeated by a web of interrelated return effects. Substantial computational power is needed to disentangle and model these return regularities.

# THE EVOLUTION OF INVESTMENT PRACTICE

Before the 1970s, the investment norm was security analysis and stock selection. In a traditional, compartmentalized approach, security analysts, technicians, and economists all funneled their insights to portfolio managers. The market was viewed as complex, in the sense that no single human mind could master all the knowledge needed for optimal decisionmaking. Coordinating the insights of multiple participants, however, is not a simple task. Needless to say, this approach has generally produced unsatisfactory results.

The EMH mounted a frontal assault on the traditional mode of investment management. In an efficient market, prices fully reflect all available information. With its flood of information and countless participants, the U.S. stock market was regarded by academicians as highly efficient. It was thought that no one could beat the market, with the possible exception of insiders. By the mid-1970s, the EMH had substantial empirical support, and was a central paradigm in finance.

The revolutionary concept of passive management was a natural outgrowth of the EMH. If security returns are random and unpredictable, then only a passive approach makes sense. Index funds that were introduced to the investment community in the mid-1970s soon blossomed in popularity.

BRUCE I. JACOBS and KENNETH N. LEVY are principals of Jacobs Levy Equity Management in Fairfield, N.J. (07006). An expanded version of this article is forthcoming in *Managing Institutional Assets*, edited by Frank Fabozzi, to be published by Ballinger Publishing, and it also forms the basis for *A Revolution in Common Stock Management: Exploiting Market Inefficiencies and Forecasting Security Returns*, by Jacobs and Levy, to be published by Dow Jones-Irwin.

Since the late 1970s, though, there has been a proliferation of empirical results uncovering security pricing patterns, or return regularities. In fact, many of these effects have long been part of market folklore. These include the low P/E, small-firm, and January effects.

Thomas Kuhn, the scientific historian, refers to such evidence of departure from conventional theory as "anomalies." In his words, "discovery commences with the awareness of anomaly, i.e., with the recognition that nature has somehow violated the paradigm-induced expectations that govern normal science" [1970, p. 52]. In recent years, investment theory has been undergoing such a process of discovery.<sup>4</sup>

At first, academics rallied to defend the EMH. Tests of market efficiency are joint tests of the effect studied and the validity of the asset pricing model used to adjust for risk. Perhaps anomalies were due solely to deficiencies in risk measurement. Yet anomalies have been shown to be robust to asset pricing models, including the CAPM and Arbitrage Pricing Theory (APT). By the early 1980s, there were undeniable chinks in the armor of the EMH.

Investors have also sought to benefit from market anomalies by using simple rules, such as buying low P/E stocks. Others have tilted toward smaller-size or higher-yielding stocks. These investors consider the stock market an ordered system; they believe that simple rules will provide consistent and predictable returns.

What has recently become evident, however, is that the market is not a simple, ordered system. In a number of instances, we have documented a pervasive and complex web of interrelated return effects. This web must first be disentangled to allow us to distinguish real effects from mere proxies. Moreover, some return effects do not produce consistent rewards. Thus, the optimal investment strategy is not as simple as tilting toward yesterday's anomalies.

Nevertheless, the indexers' nihilistic view of the market as a random system is unjustified. The market is not random, but rather complex. Computational systems can be designed to grapple with its complexity. Besides being objective and rigorous, such systems are also fully coordinated, unlike the more traditional compartmentalized approaches. Beneath the complexity of the market lie enormous inefficiency and substantial investment opportunity.

# WEB OF RETURN REGULARITIES

Figure 1 displays some interrelated return effects. The various connections shown between pairs of effects have been reported by previous studies.<sup>5</sup> For example, the small-size effect and the January



effect are related, as it has been claimed that much of the annual outperformance of small stocks occurs in the month of January. The small-size and low P/E effects also are related. Because stocks with lowerthan-average P/E ratios tend to be smaller in size, a natural question arises as to whether the size effect and P/E effect are two separate forces, or merely two different ways of measuring the same underlying phenomenon.

Many researchers have addressed this issue by examining two return effects jointly. Some conclude that the superior performance of small capitalization stocks relates to their tendency to have lower P/E ratios, while others find that low P/E stocks outperform simply because they are smaller in size. Still another viewpoint maintains that neglected securities outperform, and that low P/E and small size both proxy for this underlying effect.

While some previous academic studies have examined two or three return effects simultaneously, their findings often conflict with one another. This arises from the use of different methodologies, different time periods, and different company samples. But more fundamentally, conflicting results arise from failure to disentangle other related effects. Only a joint study of return effects in a unified framework can distinguish between real effects and illusory ones.

Consider the determinants of an individual's blood pressure. A medical researcher would not limit the analysis arbitrarily to just one or two explanatory variables, such as age and weight. More accurate evaluation can be obtained by including additional variables, such as exercise and diet. Of course, all these measures are somewhat correlated with one another. But they may all have independent predictive content.

The same holds true for the stock market: Many forces affect stock returns; some of them may be correlated, but considering only a few can produce highly misleading results.

## DISENTANGLING AND PURIFYING RETURNS

The standard approach to measuring a return effect, such as low P/E, first screens for a set of stocks below a given P/E ratio, or selects the lowest quintile of stocks as ranked by P/E. Portfolio returns are then calculated and compared to those of the universe. Any differences are ascribed to the low P/E effect. But, a low P/E portfolio by its nature will be biased unintentionally toward certain related attributes, such as higher yield, and show heavy representation in certain industries, such as utilities. Screening or quintiling procedures consider only one attribute at a time, while assuming that related effects do not matter at all. We refer to the returns produced by such methods as "naive."

The low P/E effect, measured naively, is contaminated by other forces. An oil price shock or an accident at a nuclear power plant, for instance, will have a major impact on utilities, which will be reflected in the returns of the low P/E portfolio. While fundamentals such as oil prices have no intrinsic relationship to the low P/E effect, they can confound its naive measurement.

In two papers we have introduced the alternative approach of disentangling and purifying return effects [ICFA, 1988, and FAJ, May/June 1988]. "Pure" return attributions result from a simultaneous analysis of all attribute and industry effects using multiple regression. Returns to each equity characteristic are purified by neutralizing the impact of all other effects. For example, the pure payoff to low P/E is disentangled from returns associated with related attributes, such as higher yield.

Conceptually, the pure return to low P/E arises from a lower P/E portfolio that is market-like in all other respects; that is, it has the same industry weights and the same average characteristics, such as yield and capitalization, as the market. Hence, any differential returns to such a portfolio must be attributable to the low P/E characteristic, because it is immunized from all other exposures that might contaminate returns.

#### ADVANTAGES OF DISENTANGLING

The pure returns that arise from disentangling eliminate the proxying problems inherent in naive returns. The unique insights from studying pure returns have many practical benefits for investment management.

When we distinguish between real effects and proxies, we find that some closely related effects are in fact distinct of one another. For instance, small size, low P/E, and neglect exist as three separate return effects in pure form. Each should be modeled individually, which provides greater explanatory power.

Conversely, some naive return effects merely proxy for one another, and vanish in pure form. Half of the outperformance of small stocks, for example, is reported to occur in January. But the small-firm effect, measured naively, arises from a bundle of related attributes. Smaller firms tend to be more neglected, and informational uncertainty is resolved at year-end as these firms close their books. This yearend reduction in uncertainty might induce a January seasonal return. Furthermore, smaller firms tend to be more volatile and are more commonly held by taxable investors, so they may be subject to heavier year-end tax-loss selling pressure. The abatement of selling pressure in January may lead to a price bounceback.

We find the January small-firm seasonal vanishes when measured properly in pure form. Purifying the size effect of related characteristics, such as tax-loss selling, reveals the January size seasonal to be a mere proxy. The optimal investment approach models the underlying causes directly. Because not all small firms benefit from tax-loss rebound, a strategy that directs the purchase of smaller firms at yearend is only second-best.

While we find some return effects to be real, and others to be illusory, we also find the power of some pure return effects to exceed their naive counterparts by far. This is true, for example, of the return reversal effect. This effect represents the tendency of prices to overshoot and then correct, hence the term "reversal." Yet if a jump in price is due to a pleasant earnings surprise, the superior performance will persist and not reverse. Hence, disentangling return reversal from related effects, such as earnings surprise, results in a stronger, more consistent reversal measure.

Disentangling also reveals the true nature of the various return effects. For example, low P/E stocks are usually considered defensive. But pure returns to low P/E perform no differently in down markets than in up markets. The defensiveness of low P/E in naive form arises because it proxies for defensive attributes, such as high yield, and defensive industries, such as utilities. In fact, low P/E stocks are not the safest harbor in times of uncertainty. Rather, low P/E is an imperfect surrogate for truly safe havens, such as higher yield.

Additionally, pure returns are more predictable than their naive counterparts. Pure returns possess cleaner time-series properties because they are not contaminated by proxying. For example, a time series of naive returns to the low P/E effect is buffeted by many extraneous forces, such as oil price shocks to low P/E utility stocks. In contrast, pure returns are immunized from such incidental forces, and thus can be predicted more accurately.

A major benefit of disentangling is that pure return effects avoid redundancies, and hence are additive. This allows us to model each return effect individually, and then to aggregate these attribute return forecasts to form predicted security returns. Moreover, by considering a large number of return effects, we obtain a very rich description of security pricing.

# EVIDENCE OF INEFFICIENCY

Previous research on market anomalies taken one at a time has not added to the weight of evidence contravening market efficiency. That is, if the size, P/E, and neglect effects, all measured naively, proxy for the same underlying cause, they all represent "photographs" of the same anomaly taken from different angles. We have documented, however, the existence of many contemporaneous "pure" return effects. These separate photographs of many distinct anomalies, all taken from the same angle, constitute the strongest evidence to date of market inefficiency.

Calendar-related anomalies represent additional evidence of market inefficiency. We find that return patterns such as the day-of-the-week and January effects cannot be explained by considerations of risk or value, and thus cast further doubt on the EMH [FAJ, November/December 1988].

Return effects are also contrary to current asset pricing theories, such as the CAPM, the multi-factor CAPM, and the APT. For example, the CAPM posits that systematic risk, or beta, is the only characteristic that should receive compensation. Other considerations, such as a firm's size, or the month of the year, should be unrelated to security returns.

Figure 2 displays cumulative pure returns to beta in excess of market returns for the years 1978 through 1987. These returns derive from a one crosssectional standard deviation of exposure to high beta, roughly equivalent to a sixteenth percentile ranking. While in the early years the beta attribute provided positive returns, its returns were negative thereafter. These pure returns may differ from other studies, because of our control for related attributes such as price volatility. The fact that pure returns to beta did

FIGURE 2 CUMULATIVE RETURN TO BETA



not accumulate positively over the period from July 1982 to August 1987, one of the strongest bull markets in history, casts serious doubt on the CAPM.

The existence of return effects also poses a challenge to the multi-factor CAPM.<sup>6</sup> Even the APT cannot account for the existence of several market anomalies. In fact, it appears doubtful that any meaningful definition of risk is as transient as some return effects. Thus, the weight of recent empirical evidence has buried the EMH. Also, while current asset pricing theories may contain elements of truth, none is fully descriptive of security pricing.

# VALUE MODELING IN AN INEFFICIENT MARKET

In a reasonably efficient market, prices tend to reflect underlying fundamentals. An investor superior at gathering information or perceiving value will be suitably rewarded.

In an inefficient market, prices may respond slowly to new information and need not reflect underlying fundamentals. Given the substantial evidence of market inefficiency, the efficacy of value modeling is an open question. We have examined this issue by exploring the quintessential value model the Dividend Discount Model (DDM) [FAJ, July/August 1988, and ICFA, 1989].

We find the DDM to be significantly biased toward stocks with certain attributes, such as high yield and low P/E.<sup>7</sup> In fact, some have argued that the only reason such attributes have positive payoffs is because they are highly correlated with DDM value. Further, they maintain that a properly implemented DDM will subsume these return effects.

We test this notion directly by incorporating a DDM in our disentangled framework. We find the DDM's return predictive power to be significantly weaker than that of many other equity attributes.

Hence, return effects such as P/E are not subsumed by the DDM. Rather, equity attributes emerge important in their own right, and the DDM is shown to be but a small part of the security pricing story.

The DDM embodies a particular view of the world, namely "going concern" value. But there are other sensible notions of value. For instance, current yield is an important consideration for endowment funds with restrictions against invading principal. Such endowments may be willing to pay up for higher-yielding stocks. And, in today's market environment, breakup value and leveraged buyout value have taken on increased significance. Thus, there are several competing and legitimate notions of value.

Also, we find the efficacy of value models varies over time, and often predictably. For instance, the effectiveness of the DDM depends on market conditions. Because the DDM discounts future dividends out to a distant horizon, it is a forward-looking model. When the market rises, investors become optimistic and extend their horizons. They are more willing to rely on DDM expectations. When the market falls, however, investors become myopic, and prefer more tangible attributes such as current yield.

In a price-inefficient market, the blind pursuit of DDM value is a questionable approach. Moreover, other value yardsticks clearly matter. We find that some rather novel implementations of value models offer substantial promise.

### **RISK MODELING VERSUS RETURN MODELING**

While the existence of anomalies remains a puzzle for asset pricing theories, substantial progress has been made in the practice of portfolio risk control. In recent years, several equity risk models have become commercially available. Some are APT-based, and rely on factors derived empirically from historical security return covariances. These unnamed factors are sometimes related to pervasive economic forces.

Another, perhaps more common, approach relies on prespecified accounting and market-related data. Intuitive notions of risk, such as arise from company size or financial leverage, are first identified. Then, composite risk factors are formed by combining a number of underlying fundamental data items selected to capture various aspects of that type of risk. One well-known system, for instance, defines a successful firm risk factor in terms of historical price, earnings, dividend, and consensus expectational data.

Multi-factor risk models work quite well for risk measurement, risk control (portfolio optimization), and related tasks, such as performance analysis. Both APT and composite factors are fairly stable over time. This is desirable, because meaningful definitions of a firm's risk do not change from day to day. Hence, such measures are eminently sensible for risk modeling purposes.

However, we find that the various components of composite factors often behave quite differently. For instance, each of the components of the successful company risk factor has a unique relationship to security returns. While historical relative price strength exhibits a strong January seasonal (because historical price weakness proxies for potential tax-loss selling), other fundamental components, such as earnings growth, have no seasonal pattern. Rather than combining these measures into one composite factor, we can model them more effectively individually.

Moreover, effects like return reversal and earnings surprise are ephemeral in nature, and thus unrelated to firm risk. Yet, they represent profitable niches in the market. These return-generating factors must be modeled individually, because their information content would be lost through aggregation. Hence, disaggregated measures are superior for return modeling. The use of numerous and narrowly defined measures permits a rich representation of the complexity of security pricing.

# PURE RETURN EFFECTS

We find that pure returns to attributes can be classified into two categories. The distinction is best shown graphically. Figure 3 displays cumulative pure returns in excess of the market to the return reversal and small-size effects for the period 1978 through 1987.<sup>8</sup> Clearly, return reversal provides very consistent payoffs, while the small-size effect does not. Our classification system relates not only to the consistency of the payoffs, but also to the inherent nature of the attributes. This will become apparent shortly.

The pure payoff to return reversal is remarkably powerful. It provided a cumulative return, gross of transaction costs, of 257% in excess of the market, and "worked" in the right direction over 95% of the time. We refer to these market niches that produce persistent rewards as "anomalous pockets of inefficiency" (APIs), because they are anomalous to the EMH and represent instances of opportunity.

API strategies can require very high portfolio turnover, because the particular stocks exhibiting the desired characteristics change constantly. Such strategies include purchasing recent laggards to capture return reversal, or emphasizing stocks with recent pleasant earnings surprises.

We suggest exploiting these effects as trading overlays, because no additional transaction costs are



incurred if trades are to be made regardless. For instance, an investor purchasing energy stocks would benefit by focusing on recent laggards. Moreover, APIs such as return reversal can be exploited even more effectively with real-time trading strategies. APIs appear to be psychologically motivated, as we illustrate below.

The pure payoff to the smaller size attribute illustrates the second type of return effect. Unlike APIs, the payoffs to smaller size are not consistent. For instance, the pure returns were positive in 1983, but negative in 1986. While such effects are not regular to the naked eye, they are regular and predictable in a broader empirical framework, with the use of macroeconomic information. Hence, we refer to them as "empirical return regularities" (ERRs).

As characteristics such as size are fairly stable over time, directly exploiting ERRs requires less turnover than following an API strategy. Nonetheless, optimal exploitation of ERRs, such as the size effect, still requires portfolio turnover, because small stocks should be emphasized at times and large stocks at other times.

#### ANOMALOUS POCKETS OF INEFFICIENCY

Return reversal relates to the concept of "noise" in security prices, that is, price movements induced by trading unrelated to fundamentals. The return reversal effect has psychological underpinnings. Investors tend to overreact to world events and economic news, as well as to company-specific information. Moreover, technical traders exacerbate price moves by chasing short-term trends. These types of behavior lead to overshooting and subsequent reversion in stock prices.

Another API relates to the earnings estimate revisions of Wall Street security analysts. We refer to this as the "trends in analysts' earnings estimates effect," for reasons that will soon become apparent. Upward revisions in a stock's consensus earnings estimates generally are followed by outperformance, as are downward revisions by underperformance.

The trends in estimates effect may be attributable in part to slow investor reaction to earnings estimate revisions. But it also relates to the psychology of Wall Street analysts, specifically to their herd instinct. When leading analysts raise their earnings estimate for a stock, clients will buy. Secondary analysts will then follow suit, and there will be more buying pressure.

Also, individual analysts tend to be averse to forecast reversals. Suppose an analyst had been forecasting \$2 of earnings per share, but now believes the best estimate to be \$1. Rather than admitting to a bad forecast, the analyst often shaves the estimate by a nickel at a time and hopes no one notices.

These psychological factors give a momentum to earnings revisions. Upward revisions tend to be followed by additional revisions in the same direction. The same is true for downgrades. This persistence of estimate revisions leads to a persistence in returns.

The earnings surprise effect closely relates to the trends in estimates effect. Stocks with earnings announcements exceeding consensus expectations generally outperform, and those with earnings disappointments underperform. This API relates to the tendency for earnings surprises to repeat in the following quarter. Also, we find evidence of anticipatory revisions in analysts' estimates up to three months ahead of an earnings surprise, and reactive revisions up to three months subsequent to a surprise, so there is an interplay between earnings revisions and earnings surprises.

Another analyst bias is a chronic tendency to overestimate the earnings of growth stocks. Such optimism leads, on average, to negative surprises, or "earnings torpedoes." Conversely, stocks with low growth expectations tend, on average, to produce pleasant surprises. This analyst bias arises from cognitive misperceptions. Analysts place too much emphasis on recent trends, and consistently underestimate the natural tendency toward mean reversion. For instance, during the energy crunch in the early 1980s, many analysts predicted that oil prices would continue to rise unabated.

Year-end tax-loss selling pressure also has psychological underpinnings. We find evidence of taxloss taking in depressed stocks near year-end, and the proceeds are often "parked" in cash until the new year. The abatement of selling pressure, combined with the reinvestment of the cash proceeds, produces a bounceback in January. Investors often defer selling winners until the new year, thereby deferring tax-gain recognition. This exerts downward pressure on winners in January.

But, waiting until year-end to take losses is not optimal. Before the 1986 Tax Reform Act, the optimal tax-avoidance strategy was to realize losses short-term throughout the year, prior to their becoming longterm, because short-term losses sheltered more taxable income. Yet investors are loath to admit mistakes and often defer loss-taking until year-end, when tax planning can be used as an excuse for closing out losing positions.

We find long-term tax-loss selling pressure to be stronger than short-term, which is surprising, given the greater tax-sheltering provided by shortterm losses. But it is understandable in light of the investor disposition to ride losers too long in hopes of breaking even. Investor psychology thus leads to various predictable return patterns at the turn of the year.

The turn-of-the-year effect does not arise solely from tax-motivated trading. Institutional investors often dump losers and buy winners prior to year-end to "window-dress" their portfolio. Window-dressing is not sensible from an investment viewpoint, but may serve to deflect embarrassing questions at the annual review.

# **EMPIRICAL RETURN REGULARITIES**

While APIs provide persistent payoffs, ERRs, like the size effect, do not. Nevertheless, we find these effects predictable in a broader framework, with the use of macroeconomic information.

Market commentators regularly discuss the "numbers that move the market." The focus in the early 1980s was on the money supply. Today, the emphasis is on the trade deficit and foreign exchange rates. Clearly, the stock market is driven by macro-economic news. Moreover, macroeconomic events

drive returns to some equity attributes.

Consider the linkage between foreign exchange rates and the size effect. The recent and substantial Japanese investments in U.S. stocks generally have been concentrated in more esteemed, bigger companies such as IBM and Coca-Cola. Fluctuations in the dollar/yen exchange rate alter the attractiveness of U.S. stocks to Japanese investors, which affects investment flows, thereby inducing a return differential between large and small companies.

The size effect is strongly linked to the default spread between corporate and government yields. The default spread, a business cycle indicator, widens as business conditions weaken and narrows as the economy strengthens. Smaller companies are especially susceptible to business cycle risk, as they are more fragile, less diversified, and have tighter borrowing constraints than larger firms. We find small stocks perform better when business conditions are improving; the converse is true as well. Hence, the default spread is a useful macro driver for predicting the size effect.

#### MODELING EMPIRICAL RETURN REGULARITIES

We can illustrate the predictability of ERRs by discussing the size effect in greater detail. We utilize pure returns to smaller size, thereby avoiding the confounding associated with other cross-sectional and calendar effects related to size.

We consider a variety of forecast techniques, as they pertain to the size effect, and utilize several statistical criteria for measuring "out-of-sample" forecast accuracy [FAJ, 1989]. That is, we estimate our models over a portion of the historical time series, leaving a more recent holdout sample for testing predictions. This differs fundamentally from "in-sample" data fitting.

We have categorized the size effect as an ERR, which suggests that predictive models should utilize macroeconomic drivers. Thus univariate forecasting techniques, which model only the historical returns to the size effect, are inappropriate.

Multivariate time series techniques can take explicit account of the macroeconomic forces that drive the size effect. Multivariate approaches, like vector autoregression (VAR), model a vector, or group, of related variables. A joint modeling permits an understanding of the dynamic relationships between the size effect and macroeconomic variables.

We constructed a monthly VAR model of the size effect using six economic measures as explanatory variables: 1) low-quality (BAA) corporate bond rate, 2) long-term Treasury bond rate, 3) Treasury bill rate, 4) S&P 500 total return, 5) Industrial Production Index, and 6) Consumer Price Index. We chose these macro drivers because of their importance in security valuation. Other considerations, such as the dollar/ yen exchange rate, may be helpful in modeling the size effect, but we limited our investigation to these six valuation variables.

While we found the VAR model to fit the size effect quite well in-sample, it provided poor forecasts out-of-sample. Because it has a large number of coefficients available to explain a small number of observations, a VAR model can explain historical data well. But it is likely to "overfit" the data. That is, it will fit not only systematic or stable relationships, but also random or merely circumstantial ones. The latter are of no use in forecasting, and may be misleading.<sup>9</sup>

One solution to the overfitting problem of vector time series approaches is to incorporate economic theory. Such structural econometric models include only those variables and relationships suggested by theory. Simple theories, however, are no more descriptive of the economy than they are of the stock market, and structural models generally have not performed well. An alternative solution involves a novel Bayesian technique.

## **BAYESIAN RANDOM WALK FORECASTING**

Many economic measures are difficult to predict, but their behavior can often be approximated by a random walk. A random-walk model for interest rates assumes it is equally likely that rates will rise or fall. Hence, a random-walk forecast of next month's interest rate would be simply this month's rate of interest.

That it is difficult to predict stock returns is no secret. But stock prices, like other economic data, can be approximated by a random walk. As early as 1900, Bachelier proposed a theory of random walks in security prices. A random walk is thus an eminently sensible first approximation, or "prior belief," for modeling security returns.<sup>10</sup>

Prior beliefs about the coefficients of a forecast model can be specified in many ways. One Bayesian specification imposes a random-walk prior on the coefficients of a VAR model. This prior belief acts as a filter for extracting signals (meaningful relationships in the data), while leaving accidental relationships behind. Such a specification results in a powerful forecasting tool.

The results of modeling the size effect with a Bayesian random-walk prior belief are displayed in Figure 4. The upper chart shows cumulative pure returns to small size for the period January 1982 through December 1987. The lower chart shows "out-of-sample" return forecasts for one month ahead. The fore-



casts for small stocks are positive during the early years when small stocks performed well; they gradually decline and turn negative during the last two years, as small stocks faltered.

Moreover, the Bayesian model forecasts have statistically significant economic insight. Also, the results are quite intuitive. For instance, we find that smaller firms falter as the default spread between corporate and Treasury rates widens.

#### CONCLUSION

The stock market is a complex system. Simple rules, such as always buy smaller capitalization stocks, clearly do not suffice. At the same time, the nihilism of indexing is equally unjustified.

Proper study of the market requires the judicious application of computational power. Disentangling reveals the true cross-currents in the market. Only by exposing the underlying sources of return can we hope to understand them. And only through understanding can we hope to model and exploit them.

#### REFERENCES

Jacobs, Bruce, and Kenneth Levy. "Anomaly Capture Strategies." Presented at the Berkeley Program in Finance Seminar on The Behavior of Security Prices: Market Efficiency, Anomalies and Trading Strategies, September 1986.

——. "Calendar Anomalies: Abnormal Returns at Calendar Turning Points." *Financial Analysts Journal*, November/December 1988, pp. 28-39.

——. "Disentangling Equity Return Regularities." In Equity Markets and Valuation Methods. Charlottesville: The Institute of Chartered Financial Analysts Continuing Education Series, 1988, pp. 36-46.

-----. "Disentangling Equity Return Regularities: New Insights and Investment Opportunities." *Financial Analysts Journal*, May/June 1988, pp. 18-43.

------. "Forecasting the Size Effect." Financial Analysts Journal, May/ June 1989.

——. "How Dividend Discount Models Can Be Used To Add Value." In Improving Portfolio Performance With Quantitative Models. Charlottesville: The Institute of Chartered Financial Analysts Continuing Education Series, 1989.

-----. "Investment Management: Opportunities in Anomalies?" Pension World, February 1987, pp. 46-47.

-----. "On the Value of 'Value'." Financial Analysts Journal, July/ August 1988, pp. 47-62.

-----. A Revolution in Common Stock Management: Exploiting Market Inefficiencies and Forecasting Security Returns. Homewood, IL: Dow Jones-Irwin, forthcoming.

——. "Stock Market Complexity and Investment Opportunity." In Frank Fabozzi, ed., *Managing Institutional Assets*. New York: Ballinger, forthcoming, 1990.

-----. "Trading Tactics in an Inefficient Market." In Wayne Wagner, ed., A Complete Guide to Securities Transactions: Controlling Costs and Enhancing Performance. New York: John Wiley, 1989.

——. "Web of 'Regularities' Leads to Opportunity." Pensions & Investment Age, March 7, 1988, pp. 14-15.

Kuhn, Thomas. *The Structure of Scientific Revolutions*, 2nd edition. Chicago: University of Chicago Press, 1970.

Pagels, Heinz. The Dreams of Reason: The Computer and the Rise of the Sciences of Complexity. New York: Simon & Schuster, 1988.

Simon, Herbert. "Rationality in Psychology and Economics." In Hogarth and Reder, eds., Rational Choice: The Contrast Between Economics and Psychology. Chicago: University of Chicago Press, 1987.

<sup>2</sup> The emerging field of catastrophe theory, or "chaos," should not be confused with randomness. Chaos theory has been applied to such diverse phenomena as the motion of smoke rings and the incidence of bank failures. In fact, chaos theory is a form of complexity. Ostensibly random behavior is sometimes well-defined by a series of non-linear dynamic equations.

An important characteristic of chaotic systems is that small changes in the environment can cause large, discontinuous jumps in the system. For instance, because the weather is chaotic, a butterfly stirring the air today in Japan can produce storms next month in New York.

- <sup>3</sup> As Nobel laureate Herbert Simon has asserted, the emerging laws of economic behavior "have much more the complexity of molecular biology than the simplicity of classical [Newtonian] mechanics" [1987, p. 39].
- <sup>4</sup> Science progresses through recurring cycles of a) conventional theory, b) discovery of anomalies, and c) revolution. Anomalies in the Newtonian dynamics model, for example, were resolved in 1905 by Einstein's revolutionary theory of relativity.
- <sup>5</sup> See Table I in Jacobs and Levy [FAJ, May/June 1988] for a listing of previous studies on interrelationships.
- <sup>6</sup> Time series regressions of pure returns to attributes on market excess (of Treasury bills) returns result in significant non-zero intercepts, indicating abnormal risk-adjusted payoffs. The non-zero intercepts could be due to non-stationary risk for these attributes, but we reject this explanation based on an examination of high-order autocorrelation patterns in the pure return series. Hence, these findings are anomalous in a multifactor CAPM framework.
- <sup>7</sup> Such biases represent incidental side bets inherent in the DDM. We suggest various methods for controlling these biases in the 1989 ICFA article.
- <sup>8</sup> It has often been reported that the small-size effect peaked in mid-1983. This observation is correct for naive small size, which is a bundle of several related attributes, including low price per share and high volatility. While these attributes peaked in 1983, the pure small-size effect continued to pay off positively until 1986.
- <sup>9</sup> Vector autoregression-moving average (VARMA) models attempt to overcome the overfitting problem inherent in VAR models through a more parsimonious, or simpler, representation. But VARMA models are quite difficult to identify properly. As the number of explanatory variables increases, VARMA models face what statisticians call "the curse of higher dimensionality." In these cases, VARMA forecasting is not only extremely expensive, but also rather foolhardy.
- <sup>10</sup> Technically, a random-walk model implies that successive price changes are independent draws from the same probability distribution. That is, the series of price changes has no memory and appears unpredictable. In fact, short-run stock returns are approximated well by a random walk. However, there is some evidence of a mean reversion tendency for longer-run returns.

<sup>&</sup>lt;sup>1</sup> See Pagels [1988].